

one. And I think it is admitted that because this observational or practical geometry is wanting in our elementary mathematical teaching, geometry is generally found so difficult, so inexplicably difficult, by boys.

It does not suffice to give a child a box of geometrical solids, and let him handle them and learn their names, though this is not useless. Nor does it suffice to give him a ruler and a pair of compasses to play with; and, in fact, the more we reflect on what is required to give an interest to the observations out of which familiarity with geometrical facts is to spring, the more inevitably, it seems to me, are we led to the conclusion that practical geometry is to be taught not *per se*, but by practical work, by interesting and varied applications of geometrical methods to measure and copy actually existing things.

And this at once suggests that the elementary teaching of practical geometry should consist in the manipulation of measuring instruments, and the calculations based on these measurements, which lie at the foundation of sound scientific work. I believe that all such measurements and calculations and practical constructions as are within the range of a boy, might be profitably laid before him as his work in elementary geometry: and I believe that this kind of training would moreover be of the very highest value in preparing him for good experimental work, and for a sound appreciation of scientific methods and results.

It will be advisable, however, to go into some degree of detail in order to explain my meaning to such as are not familiar with these parts of education; and in doing so, I must confess that I have not tested these details throughout by actual experience. For we have at Rugby to deal with older boys than those whom I am now contemplating, and though we do something of the kind with our younger boys, yet it is not what I should choose if I had the control of boys' education from an earlier age. Any one who wishes to see our actual course of practical geometry can do so by ordering Kitchener's "Geometrical Note-book" from the publisher of NATURE. But it must be understood to be a stopgap, and not a complete work. (I trust the author will forgive me for saying so.)

Let a boy be furnished with a ruler, a triangle (in plain wood), a pair of compasses, and a protractor. Let him have a hard pencil, and be taught how to sharpen it. First let him draw on card a decimetre scale, divided into centimetres, and in part into millimetres. This, of course, he must copy from some trustworthy scale. Insist on this, and on every part of his work being done with great care and perfect neatness, and therefore not in ink.

These are his tools. He must proceed to measure some figures provided him for this purpose; a few triangles, quadrilaterals, &c., in wood; or figures drawn on paper, are sufficient for this purpose. Every one of his measurements is neatly written in a suitable note-book, and the figure to which they apply is drawn (freehand) therein.

The next thing to proceed to is the measurement of angles, and the expression of the result in degrees and minutes, with exercises suggested by Euclid, i. 32, and its corollaries, properties of the circle, &c., which are to be practically verified, the observed results being written down, and compared with the theoretical results.

Then the lad may go on to the practical measurement of areas, beginning of course with a rectangle, which he divides into square centimetres and millimetres; he goes through the practical proof that the area of the triangle is half that of the rectangle of equal altitude on the same base; he proves Euclid, i. 47, and iii. 35; he finds the areas of various polygons of which drawings or models are given him.

Mensuration of solids is next approached, and here probably a few rules will have to be given, by which volumes are calculated from linear measurements. But in all cases the measurements must be made by the boy himself with his compasses and scale. Any one who

pleases can show his pupils how to prove the relation between the volumes of pyramids and prisms by weighing models of suitable dimensions. The same method may easily be applied to determine approximately the area of a circle; and in this, as in some other measurements, it will be well to require an estimate of the degree of approximation attained, and a mean to be taken of several measurements.

If more applications are wanting, the use of co-ordinates to express position may be explained, and some examples may be given of their application in simple problems, such as to make a plan of a room or of a garden, the scale being specified; and to copy a drawing, such as the sun with a group of spots. More advanced work to any amount will be offered by projections. The boy would be required to draw the projections of the various regular solids given to him, and perform the usual exercises of geometrical drawing. The construction by ruler and compasses of exact copies of triangles and other figures may be introduced almost anywhere, and a clear statement given of the different data from which a triangle can be constructed.

I wonder what "A Father" and mathematical teachers say to these suggestions. It will be obvious that they do not aim at making a boy a rapid analyst, or an expert problem solver; but I hope it is equally obvious that they are really calculated to make a careful and exact worker, one who shall attach precise meaning to his words, and shall be capable of using his head and hands in combination with one another in practical problems. The method is, moreover, applicable to a class, as well as to an individual pupil, and involves a very trifling expenditure on materials.

When some such course of practical geometry has been gone through, a boy may begin any scientific or deductive geometry; he had better read whatever book is read in the school to which he is going. A boy so prepared will find Euclid easy enough, but rather unaccountably indirect and clumsy; but he may be fortunate enough to be going to a school which has adopted some better arranged text book. In a year or two there will be better modern text books than now exist. Whatever book he reads, he ought to work many examples, and do original work. It is not a bad plan to give him the enunciations alone, and let him discover the proofs as far as he can. Perhaps the best text book now existing is the "Eléments de Géométrie," par Ch. Briot.

My remarks have run to considerable length, much greater than I intended, and I can apologise for it only on the ground that many teachers are thinking of the question handled in it, and that it is only by imparting our notions and our experience to one another that we shall improve our methods. I sincerely hope that "A Father's" letter to you may elicit answers from teachers more experienced and successful than I am.

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ON FRESH DISCOVERIES OF PLATYCNEMIC MEN IN DENBIGHSHIRE

IN the course of 1869 I had the good fortune to discover and explore a sepulchral cave at Perthi Chwareu, a farm about fourteen miles north of Corwen, and high up in the region of hills. It contained fifteen or sixteen skeletons, some of which were buried in a sitting posture, of ages varying from infancy upwards, and associated with the broken bones of animals that had been eaten, which belonged to the dog, fox, badger, horned sheep, Celtic staghorn (*Bos longifrons*), roe, stag, horse, wild boar, and domestic hog. The solitary work of art left behind by man consisted of a flint flake, but there were also fragments of *Mya truncata*, and of mussel and cockle-shells. The cave had been evidently used as a place of habitation

by some early race of men, and subsequently as a cemetery, and since the corpses had been deposited on the old inhabited surface, the human bones were more or less intermingled with those of the animals. The entrance had been blocked up with a barrier of large stones, and the interior was filled nearly to the roof with the fine red silt introduced through the crevices in the roof by the rain. The human remains, which were described by Prof. Busk in the essay on the discoveries published in the *Journal of the Ethnological Society*, January 1871, presented points of very high interest; for while the skulls were rather above than below the present average cranial capacity, some of the leg-bones were remarkable for the peculiar antero-posterior flattening or platycnemism of the shin-bones. And this flattening was caused by the prolongation of the bone in front of the inter-osseous ridge, and not in any great degree by its posterior extension, which is the distinctive feature of the tibiae found in the caves of Cro Magnon and of Gibraltar. The fact that these platycnemism leg-bones were associated with others of the ordinary forms, and for the most part belonging to the young, and probably to females, while the skulls were of the same type, proves that the character is not one of race, as M. Broca believed, but rather one peculiar to the individual and perhaps to the sex.

Subsequently, I was able to bring this interesting sepulchral cave into relation with remains of man from other parts of Denbighshire, through the courtesy of Mrs. Williams Wynn, in whose possession were a skull and several long bones obtained some years ago in a cave at Cefn, and of the same type as those from Perthi Chwareu. They were found along with the remains of sheep or goat, pig, fox, badger, and stag, and four flint flakes.

A chambered tomb at Cefn, explored in 1869 by Mrs. Williams Wynn, under the care of the Rev. D. R. Thomas and myself, and consisting of a chamber 5ft. wide and 6ft. long, which gradually contracted until it joined a passage 6ft. long and 2ft. wide, contained considerably more than twelve human skeletons buried in the sitting posture, of various ages, and presenting in some cases platycnemism tibiae. The skulls were of the same type as those from Perthi Chwareu, and some were possessed of peculiar upturned nasal bones that pointed unmistakeably in the direction of a *nez retroussé*. A few small broken flint pebbles were the only foreign matters in the tomb, which was built of large rough slabs of limestone placed on edge, and covered with capstones, and finally buried under a carnedd of loose fragments of limestone. A second chambered tomb with a passage was discovered by the Rev. D. R. Thomas in this carnedd in 1871, which was full of human remains of the same kind as those which I have mentioned, and in addition a few remains of dog, pig, sheep, and roe-deer were found. A broken flint and a round stone were also met with.

The remarkable correspondence of the human remains in the carnedd and the Cefn Cave with those of Perthi Chwareu, proves that the race of men who buried their dead in the tombs is the same as that which used the caves for its last resting-places. The stone chambers, with their low entrance and narrow passage, are indeed caves artificially made, and it is very possible that the idea of making "Ganggraben," or gallery graves, is derived from the ancient custom of living in and burying in caves.

It becomes an interesting question to ascertain the relative age of these cave dwellers and carnedd builders, who have so completely passed out of remembrance that their very name has perished. The evidence offered by the flint flakes may be at once dismissed as being valueless, because they were buried with the dead at least as late as the Roman occupation of Britain, and they merely indicate an antiquity not less than that of the conversion of the Romano-Celts to Christianity, a date which is very hotly contested at the present time. Nor does an appeal to the remains of the animals help us very much. The domestic

animals are nearly the same as those still kept in the district, and were introduced into Europe during the Neolithic age. The dog, however, so far as I know, was not usually eaten in Britain in Roman times, although it was an article of food in the Neolithic age in Switzerland and in Yorkshire. The sitting posture also of the corpses points in the Neolithic direction, as well as the correspondence of the skulls with those termed "river bed" by Prof. Huxley, and others which are undoubtedly of the newer stone age. On this evidence, therefore, the Neolithic date of these ancient dwellers in Denbighshire might be inferred with a high degree of probability.

All doubt, however, on the point has been removed by my discovery of a second cave some 300 yards removed from the first, during the exploration carried on by Mr. Lloyd, of Rhagatt, at the end of last August. Like the first it ran nearly horizontally into the rock, and was blocked up with earth and large masses of stone, and it contained the broken bones of the same animals associated with skeletons of the same type. The corpses had been buried in the sitting posture. During the first day's digging we obtained a beautiful polished axe made of greenstone, and with the edge uninjured by use, which had evidently been interred for some motive or other along with the dead, as well as a few splinters of flint, and one well-defined scraper of the same sort as those which the Eskimos use inserted into a handle of bone or antler. We added also the bear, *U. Arctos*, to the list of animals. And subsequently we met with a remarkably fine flint flake rather over three inches in length, which was in juxtaposition to a small heap of human bones belonging to one skeleton, and rested on the ancient floor of the cave that was indicated by a mortar-like mass of decayed stalagmite. There were also many fragments of a rude black hand-made pottery, composed of clay, worked up with small fragments of stone to prevent fracture while it was being subjected to the fire. Some were nearly an inch in thickness; while others ranged from a quarter to half an inch. It is of the same kind as that which is commonly met with in caves, occurring alike in Kühloch and Gailenreuth, and in Kent's Hole, being very frequently discovered in association with Neolithic remains. After clearing out the horizontal passage for a distance of 10ft, from the entrance, we found that it expanded into a chamber, of the dimensions of which we are unable to form an idea, as it was nearly full up to the roof with *débris*. The floor underneath the decomposed stalagmite consists of a tenacious gray clay, which has never yet yielded any remains either in Yorkshire, Wales, or Somerset, and is probably the result of the melting of the glaciers, the traces of which are abundant in the neighbourhood.

A third cave, running into the rock parallel with the last at a distance of 12ft., contained similar remains of man and the animals, as well as a fourth, which stands about half-way between Perthi Chwareu and those of Rhos-digre.

The interest of this discovery consists in the fact that the group of caves which has been used by a race of herdsmen in long-forgotten times as habitations and burial places, and the tombs at Cefn, must be referred to the Neolithic age. And we can now be certain that those people who have manifested the peculiar flattening forwards of the shin in Denbighshire belong to that age. It is a point also well worthy of note that the cranial capacity of these Neolithic men was not inferior to that of the average civilised man, although the ridges and processes for muscles indicated a greater physical power.

The clue to this remarkable series of caves was afforded by a small box of bones forwarded by Mr. Darwin, and obtained from the *débris* of a refuse heap in a neighbouring ridge, on which the Neolithic men happened to hold their feasts. We have by no means yet exhausted the evidence of a social state now unknown in Europe, which is presented by the caves and tumuli of Denbighshire.

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